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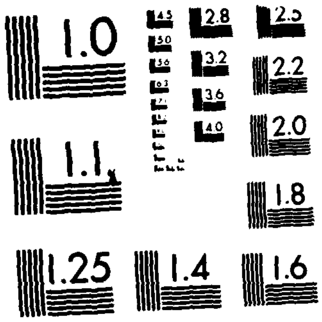

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MRC Technical Summary Report # 2662

MULTIPLE LIMIT POINT BIFURCATION FOR FREDHOLM OPERATORS OF NON-ZERO INDEX

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March 1984

(Received January 23, 1984)

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ABSTRACT

Using the method of embedding a given equation into a higher dimensional problem, examples of multiple limit point solutions emanating from a single limit point are constructed within the context of algebraic and ordinary differential equations. In both instances, the linearized problem around the limit point is assumed to lead to non-self-adjoint operators.

AMS (MOS) Subject Classifications: 58E07, 76A05

Key Words: Fredholm operators, non-zero index, bifurcation, limit point.

Work Unit Number 1 - Applied Analysis

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Sponsored by the United States Army under Contract No. DAAG29-80-C-0041.

SIGNIFICANCE AND EXPLANATION

In order to solve equations of the form

$$G(u,\lambda) = 0$$

at a limit point where $u = u(\lambda)$ ceases to be a single valued function of λ, and $du/d\lambda$ becomes unbounded, it has been a common practice in recent years to imbed the equation $G(u,\lambda) = 0$ into a 'higher dimensional' problem so that the Jacobian of the enlarged system is non-singular at the limit point. This method permits one to seek single valued solutions for $u = u(\varepsilon)$ and $\lambda = \lambda(\varepsilon)$ in terms of a new parameter ε. Despite this, it is common to speak of the solution curve passing through the limit point as exhibiting limit point bifurcation, because u is a double-valued function of λ.

Limit point bifurcations arise in numerous areas of mechanics, such as the buckling of a shallow clamped shell under uniform pressure, the flow of a non-Newtonian fluid in between rotating disks and in chemical reactor problems.

In the present paper, the possibility of two curves passing through a single limit point is discussed and is shown to depend on the dimension of the null space of the Jacobian of $G(u,\lambda)$ at the limit point, as well as the dimension of the null space of the adjoint of the Jacobian. With this method it becomes possible to study the existence of multiple limit point bifurcations in mechanics and engineering. Finally, because the theory is developed to be applicable to abstract equations, it is cast in terms of the 'Jacobian' being a Fredholm operator and its index is non-zero because the dimensions of the null spaces of the operator and its adjoint are not equal.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

MULTIPLE LIMIT POINT BIFURCATION FOR
FREDHOLM OPERATORS OF NON-ZERO INDEX

R. R. Huilgol*

1. INTRODUCTION

Let X and Y be real Banach spaces and $\lambda \in R$ be a scalar. Consider a map $G : X \times R \to Y$ and let us seek solutions of

$$G(u,\lambda) = 0 \tag{1}$$

in the vicinity of a known singular solution $(u^o,\lambda^o) \in X \times R$. Assuming G to be sufficiently smooth, we shall seek a solution "arc" $(u(\varepsilon),\lambda(\varepsilon))$, with $(u(0),\lambda(0)) \equiv (u^o,\lambda^o)$, depending on ε in $|\varepsilon| < \varepsilon_o$, in the neighborhood of (u^o,λ^o).

If we expand $G(u,\lambda)$ about (u^o,λ^o) we find that

$$\begin{aligned} G(u,\lambda) = G_u^o(u - u^o) + G^o(\lambda - \lambda^o) + \frac{1}{2} G_{uu}^o(u - u^o)(u - u^o) + \\ + G_{u\lambda}^o(u - u^o)(\lambda - \lambda^o) + \frac{1}{2} G_{\lambda\lambda}^o(\lambda - \lambda^o)(\lambda - \lambda^o) + \text{higher order terms} \quad . \end{aligned} \tag{2}$$

In (2), $G_u^o \equiv \frac{\partial G}{\partial u}(u^o,\lambda^o)$, and the other operators are similarly defined.

Let G_u^o be a Fredholm operator[1] with a null space $N(G_u^o)$, of dimension $m > 2$, and G_λ^o be such that it does not lie in the range $R(G_u^o)$ of G_u^o, i.e.,

$$G_\lambda^o \notin R(G_u^o) \quad . \tag{3.}$$

Now, because G_u^o is Fredholm,

$$X = N(G_u^o) \oplus X_1 \quad , \tag{4}$$

$$Y = R(G_u^o) \oplus Y_1 \quad , \tag{5}$$

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[1] If G_u^o is bounded, the argument that follows is unaffected. If G_u^o is unbounded, its domain which is dense in X can be turned into a Banach space via the graph norm, and one can proceed with requisite modifications. Details are left to the reader.

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where Y_1 has the same dimension as that of the null space of the adjoint $(G_u^o)^*$ of G_u^o.

If we seek small solutions to $G(u,\lambda) = 0$ when $G(u,\lambda)$ has the expansion (2), it is clear that we are led to:

$$(u - u^o)(\varepsilon) = \varepsilon\phi + \varepsilon^2 w(\varepsilon) \quad , \tag{6}$$

$$(\lambda - \lambda^o)(\varepsilon) = \varepsilon^2 \zeta(\varepsilon) \quad , \tag{7}$$

where $\phi \in N(G_u^o)$, $w \in X_1$, $\zeta \in R$. As (7) implies, on every solution arc bifurcating from (u^o,λ^o), $d\lambda/d\varepsilon = 0$ at $\varepsilon = 0$ [1,2].

Following Keller [1], and Decker and Keller [2], Huilgol [3] showed that the solutions (6) - (7) are of the "multiple limit point bifurcation" type provided G_λ^o is bounded and

$$R(G_u^o) \oplus R(G_\lambda^o) = Y \quad , \tag{8}$$

i.e.,

$$R(G_\lambda^o) = Y_1 \quad . \tag{9}$$

Now, a glance at equations (2) - (3) will show that because $G_\lambda^o : R \to Y_1$, one has $\dim Y_1 = 1$; moreover, $\dim Y_1 = \dim N((G_u^o)^*)$ and thus the Fredholm index of G_u^o is $m - 1 \neq 0$, since $m > 2$.

In order to establish (6) - (7), one can fix $\phi \in N(G_u^o)$ and define a new map $g : R \times X_1 \times R \to Y$ through

$$g(\varepsilon,v,\xi) = \begin{cases} \dfrac{1}{\varepsilon} G(u^o + \varepsilon\phi + \varepsilon v,\ \lambda^o + \varepsilon\xi),\ \varepsilon \neq 0 \quad , & (10) \\[2ex] G_u^o(\phi + v) + G_\lambda^o \xi,\ \varepsilon = 0 \quad . & (11) \end{cases}$$

Clearly, $g(0,0,0) = 0$ because $\phi \in N(G_u^o)$.

Since (8) holds, by the implicit function theorem [4], the equation $g(\varepsilon,v,\xi) = 0$ has a unique solution $v = v(\varepsilon)$, $\xi = \xi(\varepsilon)$, for $|\varepsilon| < \varepsilon_o$, such that $v(0) = 0$, $\xi(0) = 0$. Using the smoothness of G, one can write $v(\varepsilon) = \varepsilon w(\varepsilon)$, $\xi(\varepsilon) = \varepsilon\zeta(\varepsilon)$ and recover (6) - (7).

Now, there are a number of situations as in Hopf bifurcation (Weber [5]), or in the example below when it is computationally attractive to add an extra condition to (10) - (11). For instance, if $X = R^p$, we know that we can choose v to be orthogonal to ϕ.

More generally, if G_u^o is Fredholm then there is always a linear operator (e.g., a projection operator) $C^* : X \to R^m$ such that $C^* : X_1 \to \{0\}$ and C^* is an isomorphism between $N(G_u^o)$ and R^m, since $\dim N(G_u^o) = m$.

To incorporate such a C^*, it is essential to seek the solution of $G(u,\lambda) = 0$ about (u^o,λ^o) by imbedding the original problem into a "higher dimensional" one. That is to say, we solve a second equation $g_1(\varepsilon,v,\xi) = 0$, where $g_1 : R \times X_1 \times R \to Y$, and g_1 is defined, for a fixed $\phi \in N(G_u^o)$, through

$$g_1(\varepsilon,v,\xi) = \begin{cases} \begin{cases} \frac{1}{\varepsilon} G(u^o + \varepsilon\phi + \varepsilon v, \lambda^o + \varepsilon\xi), \ \varepsilon \neq 0 , & (12) \\ C^* v, \ \varepsilon \neq 0 , & (13) \end{cases} \\ \\ \begin{cases} G_u^o(\phi + v) + G_\lambda^o \xi, \ \varepsilon = 0 , & (14) \\ C^* v, \ \varepsilon = 0 . & (15) \end{cases} \end{cases}$$

Clearly, $g_1(0,0,0) = 0$ and in order to employ the implicit function theorem so that $g_1(\varepsilon,v,\xi) = 0$ has a unique solution $(v(\varepsilon),\xi(\varepsilon))$ with $v(0) = 0$, $\xi(0) = 0$, one needs to establish that the "linear operator"

$$A \equiv \begin{bmatrix} G_u^o & G_\lambda^o \\ C^* & 0 \end{bmatrix} \qquad (16)$$

is non-singular. Note that A arises because one can write (14) - (15) symbolically as:

$$g_1(0,v,\xi) = A \begin{bmatrix} v \\ \xi \end{bmatrix} . \qquad (17)$$

It was proved in Theorem I of [3] that A is non-singular, provided

(i) $R(G_u^o) \oplus R(G_\lambda^o) = Y$,

(ii) $N(C^*) = X_1, R(C^*) = R^m$,

(iii) $\dim N(G_u^o) = m$,

(iv) $N(C^*) \cap N(G_u^o) = \{0\}$.

The above idea has obvious extensions to the case where $\lambda \in R^k$ in (1) - see Theorem II of [3]. This is not pursued here.

Now, on putting $v(\varepsilon) = \varepsilon w(\varepsilon)$, $\xi(\varepsilon) = \varepsilon\zeta(\varepsilon)$ in (6) - (7), we can write $G(u,\lambda) = 0$ in (2) as:

$$A\begin{bmatrix} w \\ \zeta \end{bmatrix} \equiv \begin{bmatrix} G_u^o & G_\lambda^o \\ C^* & 0 \end{bmatrix}\begin{bmatrix} w \\ \zeta \end{bmatrix} = -\begin{bmatrix} \frac{1}{2} G_{uu}^o \phi\phi + O(\varepsilon) \\ 0 \end{bmatrix} \tag{18}$$

where (6) and (7) are used. Since $\dim N(G_u^o) \geq 2$, the possibility exists that more than one solution pair (w,ζ) can be found by solving (18) when we let ϕ be a different linear combination of $\{\phi^1,\ldots,\phi^m\}$ which span $N(G_u^o)$.

In this paper, using this idea, we construct two examples of multiple limit point bifurcation, when $\lambda \in \mathbb{R}$.

2. ALGEBRAIC EQUATIONS

Firstly, let us consider the equations

$$G_i(\underset{\sim}{u},\lambda) = \sum_{j=1}^{3} \alpha_{ij}(u_j - u_j^0) + \psi_i(\lambda - \lambda^o) + \frac{1}{2}\sum_{j,k=1}^{3} \beta_{ijk}(u_j - u_j^o)(u_k - u_k^o) \tag{19}$$

$$+ \text{ higher order terms} = 0, \quad i = 1,2 \quad ,$$

where α_{ij} is the 2×3 matrix:

$$\alpha_{ij} = \frac{\partial G_i}{\partial u_j}(\underset{\sim}{u}^o,\lambda^o) \quad , \tag{20}$$

$\lambda \in \mathbb{R}$, ψ_i, $i = 1,2$, are the components of the vector $\underset{\sim}{\psi}$ spanning $N(\underset{\sim}{\alpha}^*)$ with $\underset{\sim}{\alpha}^*$ being the adjoint of $\underset{\sim}{\alpha}$. Clearly, $\underset{\sim}{\psi} \notin R(\underset{\sim}{\alpha})$. Now, let $\underset{\sim}{\alpha}$ have a null space of dimension 2, spanned by $\underset{\sim}{\phi}^1$ and $\underset{\sim}{\phi}^2$ respectively. Then, $\dim N(\underset{\sim}{\alpha}^*) = 1$. Now we can rewrite (19), by appealing to (18), as:

$$\left[\begin{array}{ccc|c} \alpha_{11} & \alpha_{12} & \alpha_{13} & \psi_1 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \psi_2 \\ \hline \phi_1^1 & \phi_2^1 & \phi_3^1 & 0 \\ \phi_1^2 & \phi_2^2 & \phi_3^2 & 0 \end{array}\right]\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \zeta \end{bmatrix} = -\begin{bmatrix} \frac{1}{2}\sum_{j,k=1}^{3}\beta_{1jk}\phi_j\phi_k + O(\varepsilon) \\ \frac{1}{2}\sum_{j,k=1}^{3}\beta_{2jk}\phi_j\phi_k + O(\varepsilon) \\ 0 \\ 0 \end{bmatrix} . \tag{21}$$

In (21), on the left side, we have employed the fact that $\underset{\sim}{\phi}^1$ and $\underset{\sim}{\phi}^2$ are orthogonal to $\underset{\sim}{\psi}$. Thus the lower 2×3 matrix represents C^*. On the right side of (21), we have taken $\underset{\sim}{\phi} \in N(\underset{\sim}{\alpha})$ to stand for $\underset{\sim}{\phi}^1$ or $\underset{\sim}{\phi}^2$ or a linear combination of both.

Since rank $\underset{\sim}{\alpha} = 1$, the row $[\alpha_{11}\ \alpha_{12}\ \alpha_{13}]$ is a multiple of $[\alpha_{21}\ \alpha_{22}\ \alpha_{23}]$, with a factor γ, say. Provided $\Sigma(\beta_{1jk} - \gamma\beta_{2jk})\phi_j\phi_k) \neq 0$ we get a solution $\zeta(0) \neq 0$. This follows from the fact that rank $\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \psi_1 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \psi_2 \end{bmatrix} = 2$ and thus $\psi_1 \neq \gamma\psi_2$.

Now we can have two distinct solutions for $\zeta(0)$, leading to multiple limit point bifurcation, provided (say)

$$\sum_{j,k=1}^{3} (\beta_{1jk} - \gamma\beta_{2jk})\phi_j^1\phi_k^1 \neq \sum_{j,k=1}^{3} (\beta_{1jk} - \gamma\beta_{2jk})\phi_j^2\phi_k^2 \quad . \tag{22}$$

As a specific example let

$$\alpha_{ij} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad . \tag{23}$$

Then $N(\underset{\sim}{\alpha})$ is spanned by:

$$\underset{\sim}{\phi}^1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} , \quad \underset{\sim}{\phi}^2 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} , \tag{24}$$

and the domain X of $\underset{\sim}{\alpha}$ has the decomposition:

$$\left.\begin{aligned} X \equiv R^3 &= N(\underset{\sim}{\alpha}) \oplus X_1 , \\ X_1 &= \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \end{aligned}\right\} . \tag{25}$$

Similarly, since $\underset{\sim}{\alpha} : R^3 \rightarrow R^2$, the range space Y has the decomposition,

$$Y \equiv R^2 = R(\underset{\sim}{\alpha}) \oplus Y_1 , \tag{26}$$

where $R(\underset{\sim}{\alpha})$ and Y_1 are described through

$$R(\underset{\sim}{\alpha}) = \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\} , \ Y_1 = \text{span} \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\} , \ \dim Y_1 = 1 \quad . \tag{27}$$

Moreover, $\underset{\sim}{\psi} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ spans $N(\underset{\sim}{a}^*)$ as well and does not lie in $R(\underset{\sim}{a})$.

For simplicity, let us choose

$$\left.\begin{array}{r} \beta_{111} = 1,\ \beta_{213} = 2, \\ \text{other}\ \ \beta_{ijk} = 0 \end{array}\right\} . \tag{28}$$

Then, if we take $\underset{\sim}{\phi} = \underset{\sim}{\phi}^1$ in (21) we get at $\varepsilon = 0$:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} w_1^o \\ w_2^o \\ w_3^o \\ \zeta^o \end{bmatrix} = - \begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix} . \tag{29}$$

Now, the 4×4 matrix on the left side is non-singular[1] and we get the unique solutions:

$$\left.\begin{array}{r} w_1^o = w_2^o = w_3^o = -\frac{1}{12} \\ \zeta^o = -\frac{1}{4} \end{array}\right\} . \tag{30}$$

Thus, in this case, the solution to (19) is of the form

$$\left.\begin{array}{r} (\underset{\sim}{u} - \underset{\sim}{u}^o)(\varepsilon) = \varepsilon\underset{\sim}{\phi}^1 + \varepsilon^2\underset{\sim}{w}^o + O(\varepsilon^3), \\ (\lambda - \lambda^o)(\varepsilon) = \varepsilon^2\zeta^o + O(\varepsilon^3), \end{array}\right\} \tag{31}$$

where w_i^o, $i = 1,2,3$, and ζ^o are given by (30). Had we employed $\underset{\sim}{\phi}^2$ in (21), we would get a solution set similar to (31), except that $\underset{\sim}{\phi}^1$ is replaced by $\underset{\sim}{\phi}^2$ and the new values

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[1] The usefulness of incorporating C^* in (12) - (15) becomes clear here. For, without it, we have 2 equations in (19) for w_1, w_2, w_3 and ζ. Of course, the first three are inter-related. The operator C^* makes this explicit as well as leading to a square matrix A, which is invertible.

of $\underset{\sim}{w}^{o}$ and ζ^{o} are:

$$\left.\begin{aligned} w_1^o = w_2^o = w_3^o = \frac{1}{4}\,, \\ \zeta^o = -\frac{5}{4} \end{aligned}\right\} \quad . \tag{32}$$

Hence we have constructed a pair of algebraic equations yielding multiple limit point bifurcating solutions.

Now, Decker and Keller [2], while assuming $\lambda \in \mathbb{R}$, require G_u^o to have a Fredholm index zero. Therefore, their theory is inapplicable to the example chosen here since $\dim N(\underset{\sim}{a}) \neq \dim N(\underset{\sim}{a}^{*})$, while the example follows from the theorem proved in [3].

It seems to be the simplest example of multiple limit point bifurcation, since this type of bifurcation requires [2] $\dim N(G_u^o) > 2$.

3. A DIFFERENTIAL EQUATION

Once an example such as in (19), (23) - (28) has been constructed, it is clear how to generate others. To be specific, let $u^o = u^o(x)$ belong to $X \equiv L_2(0,2\pi)$. Now, consider *the differential equation* for $u \in X$:

$$\begin{aligned} G(u,\lambda) = \frac{d^2}{dx^2}(u - u^o) + (u - u^o) + (\lambda - 1)\psi \,+ \\ + f\!\left(u - u^o,\ \lambda - 1,\ \frac{d}{dx}(u - u^o)\right) = 0 \quad . \end{aligned} \tag{33}$$

In (33), $f(0,0,0) = 0$ and f is not linear in its arguments and is as smooth as one desires. The boundary conditions associated with (33) are taken to be:

$$u(0) = u(2\pi) \quad . \tag{34}$$

Clearly, $(u^o,1)$ is a solution of (33) - (34) if $u^o \in X$ meets (34).

Now, the adjoint boundary conditions are:

$$u(0) = u(2\pi) = 0,\ u'(0) = u'(2\pi) \quad . \tag{35}$$

Hence, if we let $G_u^o \equiv \frac{d^2}{dx^2} + 1$, with the boundary conditions as given by (34), then

$$N(G_u^o) = \text{span}\,\{\sin x,\ \cos x\} \quad . \tag{36}$$

However, the adjoint of G_u^o has the form $(G_u^o)^* = \frac{d^2}{dx^2} + 1$, with (35) as its boundary conditions, and hence

$$N((G_u^o)^*) = \text{span}\,\{\sin x\} \tag{37}$$

leading to the index of G_u^o being 1. Now, while the domain $\mathcal{D}(G)$ of G contains $u \in X$ meeting (34) we have yet to specify the range $R(G)$ of G. To this end, set $Y = X \setminus \{\cos x\}$; then we put $R(G) = Y$. Finally, let the function ψ be:

$$\psi(x) = \sin x \quad . \tag{38}$$

It is now clear that (33) is similar to (19) and, therefore, multiple limit point bifurcations arise from $(u^o, 1)$ provided (cf. (22)):

$$f(\phi_1, \lambda - 1, \frac{d}{dx}\phi_1) \neq f(\phi_2, \lambda - 1, \frac{d}{dx}\phi_2) \tag{39}$$

where $\phi_1(x) = \sin x$, $\phi_2(x) = \cos x$.

4. CONCLUDING REMARKS

As mentioned in [2], there are numerous examples of limit point or turn g nt bifurcation in the literature, especially in mechanics. In this literature, it is said that there are two solutions near a turning point because in the usual $(\|u\|, \lambda)$ diagram near (u^o, λ^o), there are two values for $\|u\|$, for each $\lambda \neq \lambda^o$. In the approach adopted here, following [2,3], such a solution is assumed to represent a single limit point solution for, in terms of ε, the solution curve through (u^o, λ^o) has a unique representation. To be specific, if $\varepsilon = 0$ corresponds to (u^o, λ^o), then the values of $\varepsilon < 0$ and $\varepsilon > 0$ correspond to distinct values of (u, λ), respectively, around (u^o, λ^o); or, we have a smooth solution arc through the limit point.

What has been done in the present paper is to construct examples of double solution arcs through the limit point. In the spirit of [2,3], these are called multiple limit point bifurcations, and are believed to be the first known examples of this kind in the literature.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER #2662	2. GOVT ACCESSION NO. AD-A141 660	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Multiple Limit Point Bifurcation for Fredholm Operators of Non-Zero Index		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) R. R. Huilgol		8. CONTRACT OR GRANT NUMBER(s) DAAG29-80-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 1 - Applied Analysis
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709		12. REPORT DATE March 1984 13. NUMBER OF PAGES 9
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Fredholm operators, non-zero index, bifurcation, limit point

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Using the method of embedding a given equation into a 'higher dimensional problem', examples of multiple limit point solutions emanating from a single limit point are constructed within the context of algebraic and ordinary differential equations. In both instances, the linearized problem around the limit point is assumed to lead to non-self-adjoint operators.

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